

MINI-WORKSHOP ON TOPOLOGY AND ARITHMETIC

Website: <https://aritopo.github.io/2023-08/>

Zoom broadcast link. (The event language will be in Chinese.)

zoom: 558 118 7360 passcode: sustech123

	Aug 7, Monday	Aug. 8	Aug. 9
09:30–10:30	Gao Hui	Xiao Yu Liang Tongtong	Zhao Luming
10:30–11:00	☕ (<i>coffee break</i>)	☕	☕
11:00–12:00	Guo Jingbang	Liang Tongtong	Wang Yupeng
12:00–14:00	~ <i>lunch break</i> ~	~ <i>lunch break</i> ~	~ <i>lunch break</i> ~
14:00–15:00	Liang Tongtong	Guo Jingbang	Guo Jingbang
15:00–15:30	☕	☕	☕
15:30–16:30	Liang Tongtong	Guo Jingbang	Guo Jingbang
16:30–16:45	☕	☕	☕
16:45–17:30 (or 17:45)	Xiao Yu	Guo Jingbang	<i>Free discussion</i>

Syllabus: The main reference is [BMS19]; some parts of §8-10 might be selectively omitted.

Day 1.

- Morning.
- **Introduction (a), Gao.** Compare algebraic vs. geometric studies in p -adic Hodge theory (can use complex geometry/complex Hodge theory as motivation). Motivate necessity to study integral structures in Galois representations and in p -adic cohomology theories. Discuss Breuil-Kisin modules, Breuil-Kisin-Fargues modules and their roles in number theory; this is the main motivation of BMS's work. Briefly mention Breuil-Kisin twist.
- **Introduction (b), Guo.** State the main theorems, discuss the structure of the paper and the workshop.
- Afternoon

Date: August 1, 2023.

- **Homotopical preliminaries, Liang.** Briefly introduce the language of ∞ -categories and higher algebra. This is the foundational framework used in [NS18] and [BMS19]. Briefly mention Postnikov tower and some related homological algebra.
- **THH and its variants, Liang.** Introduce the formulation of THH, TC^- , TP and TC. The key items include the Tate construction, the S^1 -action, cyclotomic structure and Tate periodicity. Briefly mention classical Hochschild homology, its variants and their HKR filtrations.
- **Descent problem and the quasi-syntomic site I, Xiao.** Discuss flat descent; then introduce the powerful quasi-syntomic site. Briefly introduce perfectoid rings, quasi-regular semi-perfectoid rings, and their properties. [BMS19, §3-4].

Day 2.

- Morning
- **Descent problem and the quasi-syntomic site II, Xiao & Liang.** The quasisyntomic descent; explain the important fact that THH and its variants are (hypercomplete) quasisyntomic sheaves.
- **THH of perfectoid, Liang.** Based on Bökstedt periodicity and some homological properties of perfectoid rings discussed yesterday, we compute THH of perfectoid rings. Furthermore, we introduce how to use homotopy fixed point spectral sequences and Tate spectral sequences to compute TC^- and TP of perfectoid rings. We try to dig out more essential hidden relations among Tate periodicity, Bökstedt periodicity, Nygaard filtration, and Breuil-Kisin twist. [BMS19, §6].
- Afternoon,
- **Motivic filtrations, Guo.** The main task is to explain the construction of the motivic filtrations on THH and its variants. The following topics might be touched upon: (1) the evenness of THH and its variants, more precisely, as sheaves on the site of quasiregular semiperfectoid rings, THH and its variants are even with respect to the canonical t-structure; (2) Nygaard filtration as the abutment filtration of the homotopy fixed point spectral sequence; (3) filtered derived categories; (4) unfold THH and its variants, together with their double speed Postnikov towers, from the site of quasiregular semiperfectoid rings to the site of quasisyntomic rings; (5) derived de Rham cohomology; (6) TC^- in characteristic p: the crystalline. [BMS19, §6-8].

Day 3.

- Morning
- **Crystalline Galois representations and Breuil-Kisin modules, Zhao.** Define crystalline representations. Discuss its classification by weakly admissible filtered φ -modules. Briefly discuss the construction of Breuil-Kisin modules associated to crystalline representations. State Kisin's fully faithfulness theorem. Time permitting, say a few more words on applications of these modules.
- **Prismatic cohomology, Wang.** Briefly review prismatic cohomology of Bhatt-Scholze. (possibly just quickly go through the introduction part of [BS22]).
- Afternoon.
- **Breuil-Kisin cohomology, Guo.** This might be treated with a general theory of relative THH. The following topics might be (im)possible: (1) Tate valued Frobenius and the Segal conjecture; (2) cyclotomic bases for THH; (3) flat \mathbb{S} -algebra as cyclotomic bases for THH; (4) the comparisons/specializations of the prismatic theory; (5) Breuil-Kisin cohomology through relative THH; (6) q-de Rham (\tilde{p} -de Rham) through relative THH. [BMS19, §11].

Some reading references

- **The BMS2.** The goal of this workshop is to study the article [BMS19], focusing on the interconnection between topological Hochschild homology and prismatic cohomology (in particular the Breuil-Kisin cohomology).
- **Survey & Lecture Notes.** There is a good survey [Mat22], which gives an account of [BMS19] in equal characteristic. Also a series of lecture notes emphasizing the arithmetic point of view [Mor19].
- **Prismatic Cohomology/Integral p -adic Hodge Theory.** The integral p -adic Hodge theory has its first incarnation in [BMS18], which is also a good reference on the algebraic preliminaries for (integral) perfectoid rings.
In [BS22], the integral p -adic Hodge theory is more systematically actualized as the theory of prismatic cohomology. There are corresponding lecture notes [Bha18] and [Ked21].
The current culmination is the *geometrization/transmutation* of the prismatic cohomology theory, achieved in [BL22a], [BL22b], and [Bha22].
- **Topological Hochschild Homology.** The modern realization of the theory of topological Hochschild homology is provided by [NS18], related to which there is a series of lecture notes [KN18]. There is also a modern survey [HN20] from the *handbook of homotopy theory*.
- **Algebraic K -theory.** The workshop requires no essential familiarity with algebraic K -theory. However, it would be helpful to slightly acquaint oneself with the motivic filtration for algebraic K -theory and the cyclotomic trace from algebraic K -theory to topological cyclic homology, and for this purpose the introduction of [Mat22] would suffice. See also [Mor21].
- **The Language of ∞ -Categories and Higher Algebra.** Everyone should be brave enough to manipulate the language of ∞ -categories without certainty. There are suitable surveys [Gep20] and [Gro20]. If one *has time*, the endless writings such as [Lur09], [Lur17], [Lur18], [Lur23], and so on, are recommended. *Wish* that everyone would have time to *submerge* oneself in [Gro21].

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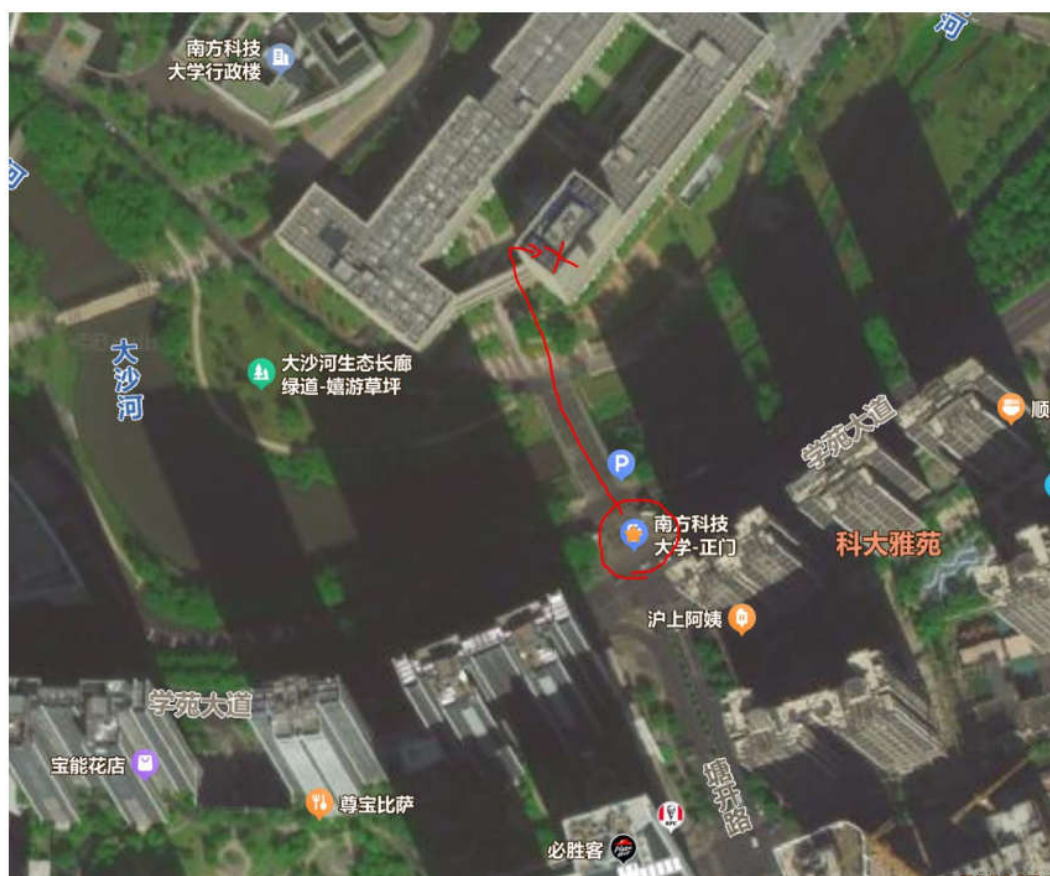
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II. The Address of the Workshop.

南方科技大学理学院大楼M1001数学系报告厅.

数学系：从1号门，也即图中正门进入。步行100米，图中打叉位置是高的塔楼。1层很容易找到报告厅。



住宿。

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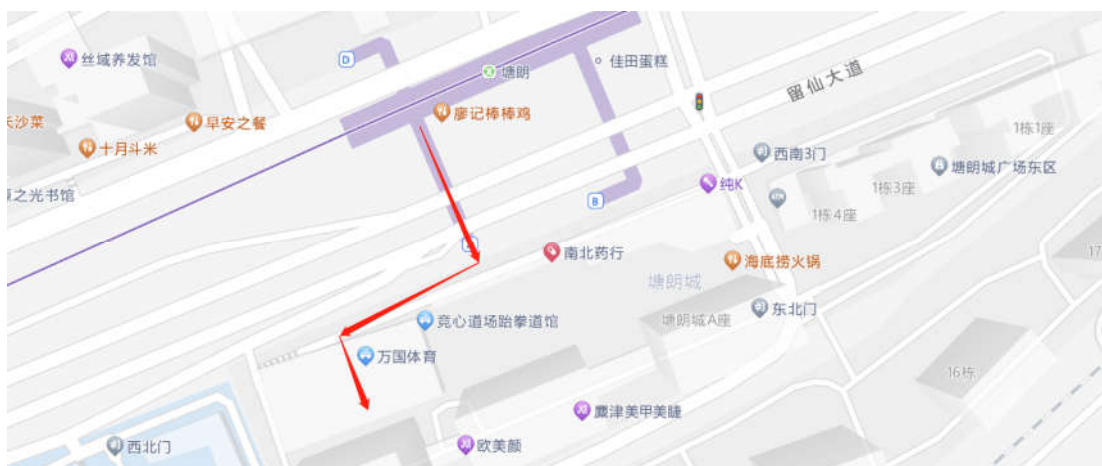
（如果是坐地铁到达，不要按照高德地图指示，而是从塘朗地铁站A出口直接进入塘朗城，往右前方走，进入乐购超市和万宁之间的走道，然后左转，可以看到君璞酒店的广告牌，广告牌左边即为酒店的电梯，乘坐电梯到7楼，7楼为前台大厅及餐厅）

入住方式：报自己姓名.说南科大数学系会议预订的即可

电话：0755-8663 9988

住宿的时候酒店要向参会者收取每人500-1000的押金（可刷信用卡预授权），以抵扣可能产生的杂费（洗衣、吃饭、房间内物品、损坏赔偿等）。

住宿**不包早餐**(因为报销标准原因)。（教师有可能有早餐，请咨询前台）



IV. Transportation.

当您到达宝安国际机场或深圳各大火车站、汽车站后，都能通过机场大巴、地铁、公交车、出租车等交通工具前往南方科技大学。推荐乘坐深圳地铁，5号线的塘朗站位于南科大一号门附近。此外，您也可以乘坐出租车来校，目前南方科技大学有四道校园门（一号门、三号门、六号门、七号门）通车。

以下是从几大客运枢纽来校的推荐路线：

深圳北站:从深圳北站乘坐地铁与公交来校都十分方便，可乘坐地铁5号线从深圳北站到塘朗站（深圳北站-长岭陂站-塘朗站），并从C出口出站，然后步行700米左右到达学校一号门。

宝安国际机场:

乘坐地铁11号线到前海湾站，然后换乘地铁5号线到塘朗站并从C出口出站，步行700米左右到达学校1号门。

V. Map.

